Práctico 8

8.01

a) $\sum_{n=1} \frac{1}{n^2}$ $\{1, 1/4, 1/9, 1/16, 1/25, \dots\}$

b) $\sum_{n=1} \frac{n}{(n+1)(n+2)}$ $\{1/6, 1/6, 3/20, 2/15, 5/42, \dots\}$

c) $\sum_{n=0} \frac{1}{2n+1}$ $\{1, 1/3, 1/5, 1/7, 1/9, \dots\}$

d) $\sum_{n=0} (-1)^{n+1} \frac{1}{n!}$ $\{-1, 1, -1/2, 1/6, -1/24, \dots\}$

8.02

a) $\sum_{n=1}^{\infty} 2\left(\frac{2}{3}\right)^n$ $S = \frac{1}{1-r} = \frac{1}{1-2/3} = 3 \cdot 2 = \boxed{6}$

b) $\sum_{n=1}^{\infty} 2\left(-\frac{2}{3}\right)^n$ $S = \frac{1}{1+2/3} = \frac{3}{5} \cdot 2 = \boxed{\frac{6}{5}}$

c) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ $S = \frac{1}{1-1/2} = \boxed{2}$

d) $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n$ $S = \frac{1}{1+1/2} = \boxed{\frac{2}{3}}$

8.03

a) $\sum_{n=0}^{\infty} \frac{3}{2^n} = \sum_{n=0}^{\infty} 3 \cdot \underbrace{\left(\frac{1}{2}\right)}_{r}^{n}$

Converge ya que $|r| < 1$ $\boxed{S = 6}$

b) $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$

Diverge ya que $|r| > 1$

c) $\sum_{n=0}^{\infty} (0,9)^n$

Converge ya que $|r| < 1$ $\boxed{S = 10}$

d) $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$

Converge ya que $|r| < 1$ $\boxed{S = 8}$

8.05

a) $\sum_{n=1}^{\infty} \frac{n}{n^2+2n+3}$

$$\lim_{n\to\infty} \frac{\frac{n}{n^2+2n+3}}{1/n} =$$

Serie Armónica: Diverge

$$\lim_{n\to\infty} \frac{n}{n^2+2n+3} \cdot n =$$

$$\lim_{n\to\infty} \frac{n^2}{n^2+2n+3} =$$

$$\lim_{n\to\infty} \frac{1}{1+\frac{2}{n}+\frac{3}{n^2}} = \boxed{1}$$ ⇒ Ambas series Divergen por ser $l > 0$

b) $\sum_{n=1}^{\infty} \frac{3n+1}{n^3-4}$

$$\lim_{n\to\infty} \frac{\frac{3n+1}{n^3-4}}{(1/n^2)} =$$ $\rightarrow$ Converge por serie armónica generalizada

$$\lim_{n\to\infty} \frac{(3n+1)n^2}{n^3-4} =$$

$$\lim_{n\to\infty} \frac{3n^3+n^2}{n^3-4} =$$

$$\lim_{n\to\infty} \frac{3+1/n}{1-4/n^3} = \boxed{3}$$ ⇒ Ambas series convergen por ser $l > 0$

c) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+1}}$

$$\lim_{n\to\infty} \left(\frac{1}{n\sqrt{n+1}} \Big/ \frac{1}{n^{3/2}}\right)$$ $\rightarrow$ Como $p = 3/2 > 1$ ⇒ Converge

$$\lim_{n\to\infty} \frac{n^{3/2}}{n\sqrt{n+1}} = \lim_{n\to\infty} \frac{n^{1/2}}{\sqrt{n+1}} =$$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+1}} = \lim_{n\to\infty} \frac{\sqrt{n}/\sqrt{n}}{\sqrt{n+1}/\sqrt{n}} =$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{1+1/n}} = \frac{1}{1} = \boxed{1}$$ ⇒ Ambas series convergen por ser $l > 0$

8.06

a) $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

$$\lim_{n\to\infty} \frac{8^{n+1}}{(n+1)!} \cdot \frac{n!}{8^n} =$$

$$\lim_{n\to\infty} \frac{\cancel{8^n} \cdot 8 \cdot \cancel{n!}}{(n+1)\cancel{n!} \cdot \cancel{8^n}} =$$

$$\lim_{n\to\infty} \frac{8}{n+1} = \boxed{0}$$ Converge por ser menor que 1.

b) $\sum_{n=1}^{\infty} \frac{5^n}{n^5}$