HOJA N°

FECHA

ALGEBRA LINEAL PREFINAL

05-12-01

① SEA $\lambda \in \mathbb{R}$ Y $f: \mathbb{R}^4 \to \mathbb{R}^4$, LA TRANSFORMACIÓN LINEAL DEFINIDA POR:

$$f(x\ y\ z\ t) = (x+y+z+t,\ (\lambda+1)y,\ -y+(1-\lambda)z-t,\ y+t)$$

DECIDIR SI f ES DIAGONALIZABLE. JUSTIFICAR LA RESPUESTA

$$A = |f|_C = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & \lambda+1 & 0 & 0 \\ 0 & -1 & 1-\lambda & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$|xI - A| = \begin{vmatrix} x-1 & -1 & -1 & -1 \\ 0 & x-(\lambda+1) & 0 & 0 \\ 0 & 1 & x-(1-\lambda) & 1 \\ 0 & -1 & 0 & x-1 \end{vmatrix} = (x-1) \begin{vmatrix} x-(\lambda+1) & 0 & 0 \\ 1 & x-(1-\lambda) & 1 \\ -1 & 0 & x-1 \end{vmatrix}$$

$$= (x-1)^2 (x-(\lambda+1))(x-(1-\lambda))$$

VALORES PROPIOS $\begin{cases} 1 & \text{MULTIPLICIDAD } 2 \\ \lambda+1 & \text{MULTIPLICIDAD } 1 \\ 1-\lambda & \text{MULTIPLICIDAD } 1 \end{cases}$

SI $\boxed{x = 1}$

$$\begin{pmatrix} 0 & -1 & -1 & -1 \\ 0 & 1-(\lambda+1) & 0 & 0 \\ 0 & 1 & 1-(1-\lambda) & 1 \\ 0 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 0 & -1 & -1 & -1 & 0 \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 1 & \lambda & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} F_2 = F_2 - \lambda F_1 \\ F_3 = F_3 + F_1 \\ F_4 = F_4 - F_1 \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & \lambda & \lambda & 0 \\ 0 & 0 & \lambda-1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} F_1 = F_1 + F_4 \\ F_2 = F_2 - \lambda F_4 \\ F_3 = F_3 + F_4 \end{array}$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad \begin{array}{l} \longrightarrow y = 0 \\ \\ \longrightarrow \lambda = 1 \\ \longrightarrow z = -t \end{array}$$

$$(x\ y\ z\ t) = (x, 0, -t, t)$$
$$= x(1\ 0\ 0\ 0) + t(0\ 0\ -1\ 1)$$

$$S_{x=1} = \langle (1\ 0\ 0\ 0), (0\ 0\ -1\ 1) \rangle$$

COMO LA MULTIPLICIDAD DEL VALOR PROPIO $1 = 2 = \text{Dim } S_{x=1}$

NOTA

ENTONCES f ES DIAGONALIZABLE PARA $\lambda = 1$
NO ME FIJO EN LA DIMENCIÓN DE LOS OTROS VALORES PORQUE LA DEFINICIÓN DE VALOR PROPIO, ME DICE QUE AL MENOS LA DIMENSIÓN ES 1, Y MAYOR A 1 NO PUEDEN SER PORQUE ESTAMOS EN $\mathbb{R}^4$

Si $\boxed{\lambda = 0}$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad F_4 = F_4 - F_3$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \to y = 0 \\ \\ \to T = 0 \\ \to z = 0 \end{array}$$

$$(x\ y\ z\ t) = (x\ 0\ 0\ 0) = x(1\ 0\ 0\ 0)$$

$$S_{x=1} = \langle (1\ 0\ 0\ 0) \rangle$$

$\Rightarrow f$ NO ES DIAGONALIZABLE PARA $\lambda = 0$

Si $\boxed{\lambda \neq 0}$ y $\boxed{\lambda \neq 1}$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad F_3 = F_3 - \lambda F_4$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\lambda & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) \quad F_4 = F_4 - (1-\lambda) F_3$$

$$\left(\begin{array}{cccc|c} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\lambda & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \to y = 0 \\ \\ \to T = 0 \\ \to z = 0 \end{array}$$

$$(x\ y\ z\ t) = (x\ 0\ 0\ 0) = x(1\ 0\ 0\ 0)$$

$$S_{x=1} = \langle (1\ 0\ 0\ 0) \rangle$$

$\therefore f$ NO ES DIAGONALIZABLE PARA $\lambda \neq 1$

NOTA