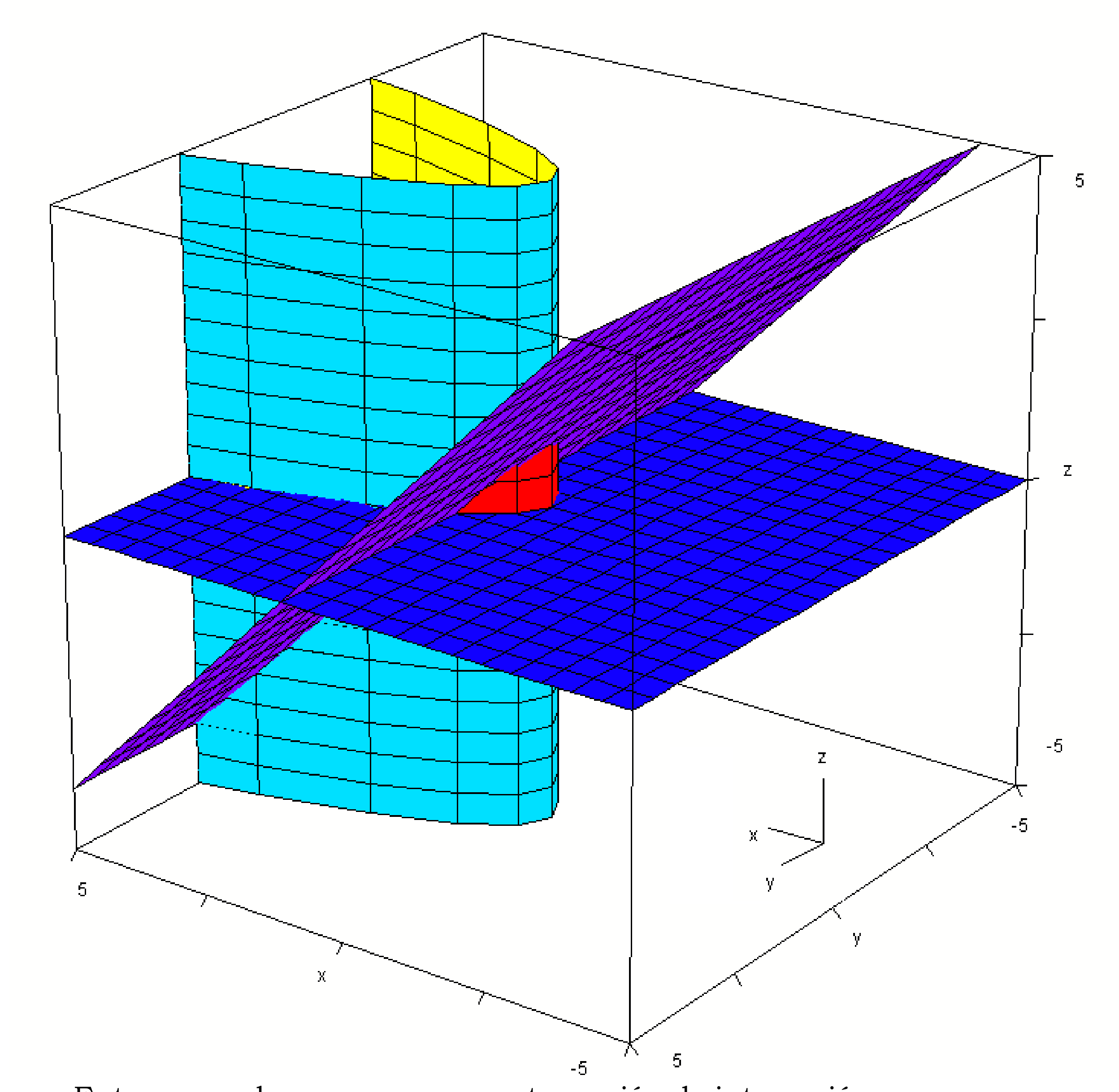
Análisis Matemático II – Final Regular

Fecha: 13/12/2005

1) Determine el volumen del sólido limitado por el cilindro $x=y^2$ **y los planos** $z=0$ **y** $x+z=1$



Entonces podemos ver que nuestra región de integración es

$$R \begin{cases} 0 \leqslant x \leqslant 1 \\ -\sqrt{x} \leqslant y \leqslant \sqrt{x} \\ 0 \leqslant z \leqslant 1-x \end{cases}$$

$$\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} dz.dy.dx \quad = \quad \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} 1-x\, dy.dx \quad =$$

$$\int_0^1 (y-xy) \Big|_{-\sqrt{x}}^{\sqrt{x}} dx \quad = \quad \int_0^1 (\sqrt{x}-\sqrt{x}\,x)-(-\sqrt{x}+\sqrt{x}\,x)dx \quad =$$

$$2\int_0^1 (\sqrt{x}-\sqrt{x}\,x)dx \quad = \quad 2\int_0^1 (x^{\frac{1}{2}}-x^{\frac{3}{2}})dx \quad = \quad 2(\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}})\Big|_0^1 \quad = \quad 2(\frac{2}{3}-\frac{2}{5}) \quad = \quad \mathbf{\frac{8}{15}}$$

2) i- Evaluar el polinomio de Taylor de 2° grado de la función $f(x,y)=x.arctg(\frac{y}{x})$ **con** $x\neq 0$ **en el punto (1,0).**

Formula de Taylor de 2° orden:

$$f(x_0+x,y_0+y)=f(x_0,y_0)+\nabla f(x_0,y_0).(x,y)+\frac{1}{2}(x,y).H(x_0,y_0).(\frac{x}{y})+R(x,y)$$

Resolviendo:

$$f(x_0+x,y_0+y)=f(1+x,y)$$

$$f(x_0,y_0)=f(1{,}0)=0$$

$$\nabla f(x_0,y_0)=(arctg(\frac{y_0}{x_0})+x_0.\frac{1}{1+\frac{y_0^2}{x_0^2}}.\frac{-y_0}{x_0^2},x_0.\frac{1}{1+\frac{y_0^2}{x_0^2}}.\frac{1}{x_0})=$$

$$(arctg(\frac{y_0}{x_0})+x_0.\frac{1}{\frac{x_0^2+y_0^2}{x_0^2}}.\frac{-y_0}{x_0^2},x_0.\frac{1}{\frac{x_0^2+y_0^2}{x_0^2}}.\frac{1}{x_0})=(arctg(\frac{y_0}{x_0})+\frac{-x_0y_0}{x_0^2+y_0^2},\frac{x_0^2}{x_0^2+y_0^2})=(0{,}1)$$

$$\frac{d^2f}{dx^2}(x_0,y_0)=\frac{1}{1+\frac{y_0^2}{x_0^2}}.\frac{-y_0}{x_0^2}-\frac{y_0(x_0{}^2+y_0{}^2)-x_0y_0.2x_0}{(x_0{}^2+y_0{}^2)^2}=\frac{-2y_0x_0{}^2-2y_0{}^3}{(x_0{}^2+y_0{}^2)^2}=0$$

$$\frac{d^2f}{dydx}(x_0,y_0)=\frac{d^2f}{dxdy}(x_0,y_0)=\frac{2x_0(x_0^2+y_0^2)-x_0^2 2x_0}{(x_0^2+y_0^2)^2}=\frac{2x_0y_0^2}{(x_0^2+y_0^2)^2}=0$$

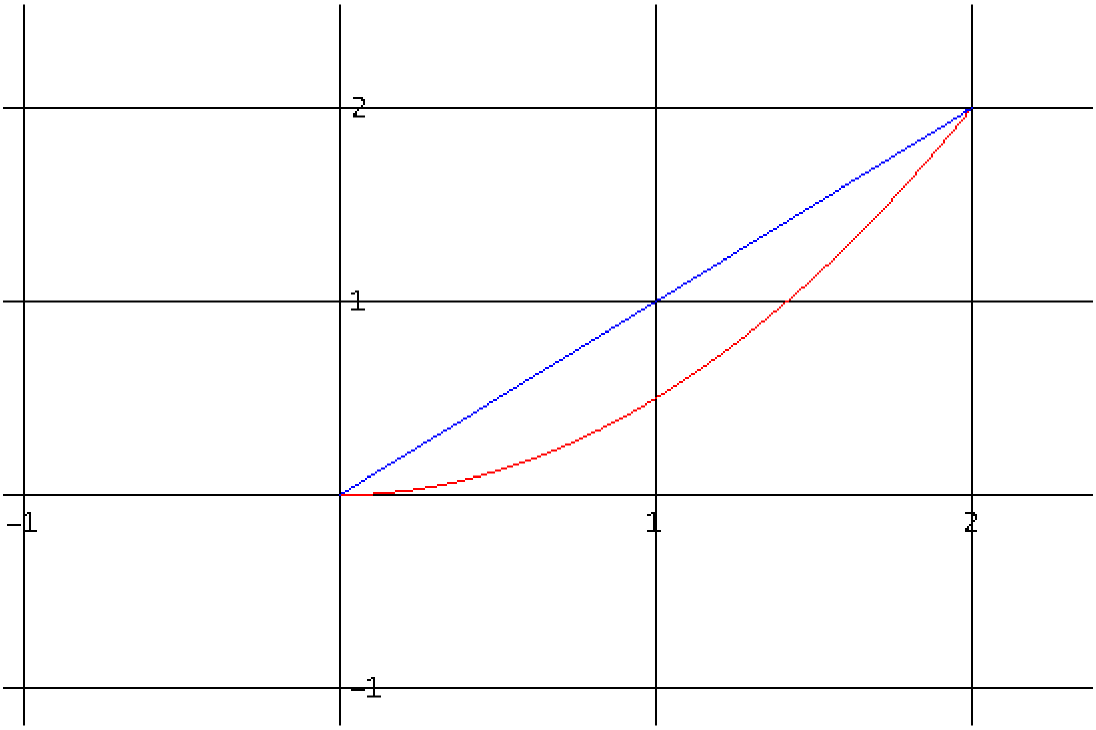
$$\frac{d^2f}{dy^2}(x_0,y_0)=\frac{-x_0^2 2y_0}{(x_0^2+y_0^2)^2}=0$$

Entonces:

$$f(1+x,y)=0+(0{,}1).(x,y)+0+R(x,y)=y+R(x,y)$$

El polinomio de Taylor de 2° orden en el punto (1,0) es **y**

3) Calcular $I=\int_c (2x-y)\,dx+(x+3y)\,dy$ **donde** $x=t$ **y** $y=\frac{1}{2}t^2$ **, y** $0\leqslant t\leqslant 2$



Utilizando Teorema de Green

$P=(2x-y)$ $\qquad \frac{dP}{dy}=-1$

$Q=(x+3y)$ $\qquad \frac{dQ}{dx}=1$

$$\frac{dQ}{dx}-\frac{dP}{dy}=2$$

$$I=\int_0^2\int_{\frac{1}{2}x^2}^{x} 2dydx \;=\; \int_0^2 2(x-\frac{1}{2}x^2)dx \;=\; 4-\frac{8}{3} \;=\; \mathbf{\frac{4}{3}}$$

Utilizando Integrales de Linea

Primer Curva:

$x=t \qquad dx=dt$

$y=\frac{1}{2}t^2 \qquad dy=tdt$

t se recorre de 0 a 2

$$\int_{c_1} =\int_0^2 (2t-\frac{1}{2}t^2)\,dt+(t+3\frac{1}{2}t^2)\,tdt$$

$$\int_0^2 (2t - \frac{1}{2}t^2 + t^2 + \frac{3}{2}t^3)\,dt =$$

$$(2^2 + \frac{1}{6}2^3 + \frac{3}{8}2^4)\,dt = \qquad \mathbf{\frac{34}{3}}$$

Segunda Curva:
$x = t = y \qquad dx = dt = dy$

t se recorre de 2 a 0

$$\int_{c_2} = \int_2^0 (2t - t)\,dt + (t + 3t)\,dt =$$

$$\int_2^0 5t\,dt = \qquad \mathbf{-10}$$

$$I = \int_{c_1} + \int_{c_2} = \qquad \frac{34}{3} - 10 = \qquad \mathbf{\frac{4}{3}}$$

4) Probar que todos los planos tangentes al cono $z^2 = x^2 a^2 + y^2 b^2$ con $z > 0$, $a \neq 0$ y $b \neq 0$ pasa por el origen.